On Discontinuities in Motor Learning:  
A Longitudinal Study of Complex Skill Acquisition on a Ski-Simulator

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ABSTRACT. The qualitative behavioral reorganizations that occurred during the acquisition of a complex motor skill were examined. Five novice participants practiced for 39 sessions of ten 1-min trials on a modified version of the ski-simulator. Analyses focused on the motion of the apparatus platform, modeled as a self-sustained oscillator. At the beginning of the experiment, all participants adopted a behavior that could be modeled with a highly nonlinear stiffness function and a Rayleigh damping function. The behavior in the final part of the experiment was captured by a qualitatively different model, with a linear stiffness function and a van der Pol damping behavior. The transition from the initial to the final model was gradual and was marked in most cases by an abrupt increase of oscillation frequency. During the transition stage, the 2 damping behaviors seemed alternately exploited within each trial. The results are discussed in the framework of the dynamical systems approach to motor coordination and learning, considering motor skill acquisition as a phase transition.

Keywords: dynamical modeling, longitudinal study, motor learning, phase transition

For a long time, motor learning has been conceived of as a progressive and continuous refining process. That basic idea was notably promoted by cognitive theories of motor control and learning. According to that point of view, the individual builds, during learning, an internal representation of the response, which is progressively refined with practice on the basis of information feedback or model presentation. The conception of learning as a continuous process has been supported by statements in favor of the classical power law, which describes the evolution of performance as a power function of the duration of practice or the number of trials carried out (Crossman, 1959; A. Newell & Rosenbloom, 1981).

The conceptions of motor learning as continuous were severely criticized by K. M. Newell (1991), who portrayed the process of learning, on the contrary, as discontinuous, nonlinear, and marked by deep, qualitative behavioral reorganizations. According to Newell, conclusions concerning continuous descriptions of learning are mainly the reflection of methodological artifacts. At first, learning experiments were generally conducted with very simple tasks, mostly restricted to a single degree of freedom, as in the linear-positioning paradigm used by Adams (1971). The requirements in terms of motor coordination were quite low and, unlike what occurs in most real-life situations, possibly did not necessitate the acquisition of new patterns. As such, continuous performance curves represented only a special case of learning.

A second argument concerned the short duration of most studies, which rarely went beyond the very first adaptation to the task and as such did not allow qualitative modifications of the behavior to appear. Third, performance was generally indexed in those experiments by errors or chronometrical measures, and such outcome indices are ill-suited to reveal possible alterations in the underlying processes during learning. Finally, the power law was generally supported by group learning curves, and the averaging process could be suspected to eliminate eventual discontinuities at the individual level.

The discontinuous conception of learning advocated by K. M. Newell (1991) was more recently supported by Zanone and Kelso (1992, 1997), who introduced a series of propositions based on the theory of nonlinear dynamical systems. According to the dynamical point of view, the coordination modes spontaneously adopted by participants in a given task are the expression of the intrinsic dynamics...
of the system. Some coordination modes appear particularly stable and easy to control and are conceived of as the attractors of the intrinsic dynamics. For example, in the bimanual oscillatory task studied by Zanon and Kelso, the in-phase and antiphase coordination modes seemed to be spontaneously adopted and to constitute two stable, fixed points in the attractor landscape characterizing the task. The intrinsic dynamics forms the basis on which learning takes place. If the pattern to be learned corresponds to one of the stable solutions of the attractor landscape, then the spontaneous coordination tendencies will constitute a resource for learning. Conversely, when the intended pattern corresponds to an unstable zone, the learner will have to fight against the initial attractors when establishing a novel coordination mode.

Zanon and Kelso (1992, 1997) showed that practicing a spontaneously unstable pattern (for example, a 90° relative-phase coordination between the hands) led to its stabilization (see also Swinnen, Dounskaya, Walter, & Serrien, 1997). In other words, learning can be conceived of as an alteration of the intrinsic dynamics of the system, with the sinking of a new stable fixed point in the attractor landscape. Note that the alteration was not confined to the practiced pattern: Zanon and Kelso (1992) showed that the performance of the symmetry pattern (i.e., the 270° relative phasing) was facilitated even though that pattern was not specifically practiced. In addition, those authors reported a transitory destabilization of one of the initial attractors, the antiphase pattern. They interpreted those qualitative alterations of the phase diagram as a phase transition.

In dynamical systems theory, a phase transition is defined as an abrupt change of the behavior of the system, determined by the evolution, beyond a critical threshold, of a so-called control parameter. In the conception of learning as a phase transition, the notion of a qualitative discontinuity between the initial behavior of the beginner and the behavior exhibited at the end of learning is emphasized. The results of some recent experiments focused on expert–novice differences in sport-like situations clearly supported that idea. As an example, Temprado, Della-Grasta, Farell, and Laurent (1997) studied the intralimb coordination of the serving arm in the volleyball service. The coordination patterns of experts and novices were found to be qualitatively different in terms of the type of shoulder–wrist coupling (in-phase versus antiphase), suggesting a discontinuity in the learning process. In a similar vein, Delignières et al. (1998) found that novices and experts differed qualitatively in their swings under parallel bars in terms of both relative phasing and frequency ratio between the two main oscillators of the system. The experiment provided a good illustration of the concept of spontaneous coordination in such gross motor, sport-like skills: All the beginners adopted the same coordination mode, characterized by the simplest frequency ratio (1:1) and relative phasing (in-phase). Moreover, the results of that experiment showed the resistance of those initial coordination modes to change: Despite 80 trials on the task, no qualitative evolution of that initial coordination mode was observed.

In a more recent article, Zanon and Kelso (1997) showed that there can be a different route to learning, depending on the initial attractor landscape and the proximity of the to-be-learned pattern to already-established attractors. In that case, learning seemed to involve a simple shift of an existing stable coordinative state in the direction of the to-be-learned pattern. For example, 2 participants in Zanon and Kelso’s experiment were initially characterized by a multistable attractor landscape, including the in-phase and antiphase patterns but also the 90° relative-phase pattern. The participants were instructed to learn a new pattern at 135° relative phase, close to the initial 90°. Results showed that learning occurred through a gradual shift of the initial 90° attractor in the direction of to-be-learned 135°. In a case such as that, learning can be conceived of not as a phase transition characterized by a qualitative change in the attractor landscape but instead as a parametric evolution of the initial coordination dynamics.

More generally, the route to learning could be different, depending on the relationship between novice and expert behaviors. The discontinuous route is likely to appear if expert behavior is qualitatively different from novice behavior, and then should be established in competition with the initial coordination dynamics. In such a case, a phase transition is expected (Zanon & Kelso, 1992) because there is no continuity between the initial and final behaviors. The qualitative differences evidenced by Temprado et al. (1997) and by Delignières et al. (1998) between novices and experts suggest that in the situations they studied, learning should follow the first route. A continuous route should occur when the expert behavior constitutes a refined adaptation of the initial behavior. Learning in the latter case should appear as a gradual change of behavior.

To our knowledge, in no experiments conducted to date has it been possible to directly observe the transition from novice to expert behavior in the acquisition of a complex skill. Changes in behavior were generally suggested by comparisons of expert and novice behavior (e.g., Delignières et al., 1998; Temprado et al., 1997) or pretests versus posttests (e.g., Cordier, Mendes Franca, Paillou, & Bolon, 1994; Sanders, 1995), but there have been few attempts to analyze the dynamics of those changes. Our aim in the present work was therefore to perform a longitudinal study of learning, with a particular focus on changes in behavior, in order to elucidate their true nature. Our concern while designing this experiment was to try to satisfy K. M. Newell’s (1991) major criticisms of most of the former learning experiments, as previously discussed. Therefore, we decided to analyze the evolution of the behavior of (initially) novice participants in a complex, sport-like task. The experiment was pursued until the evident stabilization of an expert coordination mode. We finally decided to stop the experiment after participants had had 13 weeks of practice, with three training sessions per week.
The experiment was performed with the ski-simulator, a commercially available apparatus previously used in many experiments, especially in research on skill acquisition (den Brinker, Stäbler, Whiting, & van Wieringen, 1986; den Brinker & van Hekken, 1982; Delignières, Nourrit, Deschamps, Lauriot, & Caillou, 1999; Durand, Geoffroï, Varray, & Préfaut, 1994; van Emmerik, den Brinker, Vereijken, & Whiting, 1989; Vereijken, 1991; Vereijken, van Emmerik, Bongaardt, Beek, & Newell, 1997; Vereijken, van Emmerik, Whiting, & Newell, 1992; Vereijken & Whiting, 1990; Vereijken, Whiting, & Beek, 1992; Wulf, Hös, & Prinz, 1998; Wulf & Weigel, 1997). Notably, Vereijken et al. (1997) suggested that learning on the ski-simulator proceeds in three different stages, which are characterized by qualitatively distinct coordinate structures and are interpreted as different instances of pendulum systems (balancing pendulum, hanging pendulum, and buckling compound pendulum). As a consequence, we clearly expected to observe qualitative changes in participants' behavior during the course of learning.

As pointed out by K. M. Newell (1991) and Vereijken (1991), the focus in motor learning experiments should be not on performance output (e.g., on the ski-simulator, oscillation amplitudes or frequency) but on motor coordination itself. For example, Vereijken (1991) proposed an analysis in terms of relative phase transitions between the oscillations of the apparatus and the movements of the center of mass. That approach allowed her to localize, within the oscillation of the simulator, the time of forcing (i.e., the position in the cycle at which participants began to exert a force on the platform of the apparatus). We recently developed a new approach based on the dynamical modeling of the movements of the platform of the ski-simulator (Delignières et al., 1999). In that approach, one considers the platform as an end-effector and assumes that its kinematics contains information about the overall coordination dynamics and especially about the way in which participants exert force on the apparatus in order to sustain oscillations (Vereijken, 1991; Vereijken et al., 1992).

In a number of recent studies, investigators have tried to model biological rhythmic movements as self-sustained oscillators (Beek & Beek, 1988; Beek, Rikkert, & van Wieringen, 1996; Beek, Schmidt, Morris, Sim, & Turvey, 1995; Delignières et al., 1999; Kay, Saltzman, Kelso, & Schöner, 1987; Mottet & Bootsva, 1999). Those studies were based on the assumption that the central nervous system uses limit cycle dynamics to produce rhythmic movements. The investigators' aim was to provide macroscopic models that contain a small number of parameters and capture the essential features of rhythmic movements. In that framework, rhythmic movements are modeled as oscillators that obey second-order ordinary differential equations of the following kind:

\[ m \ddot{x} + g(x) + f(x, \dot{x}) \dot{x} = 0, \quad (1) \]

where \( x \) represents position. The dot notation indicates differentiation with respect to time. The first term expresses the inertia of the system, the second the system's stiffness, and the third the system's damping (negative and positive, i.e., injection and dissipation of energy). The major concern in that approach is to identify the nonlinear stiffness and damping functions that characterize the dynamics of the movement.

Beek and Beek (1988) showed that the stiffness and the damping functions were necessarily composed of terms \( x^\alpha \dot{x}^\beta (\alpha, \beta = 0, 1, 2, 3, \ldots) \) and that a limited catalogue of such terms represents viable transformations of the harmonic oscillator (\( x + x = 0 \)). The aim of those authors was to account for the successive phases of hand motion in juggling in terms of the quantitative dynamics of a single autonomous oscillator. Developing a model by means of Chebychev-type polynomials, they showed that the continuous representation of such biological movement imposed some restrictions in the choice of the terms to include. More precisely, \( g(x) \) should be composed of terms from the Duffing series \((x^4, x^3, x^2, \ldots)\), and \( f(x, \dot{x}) \) should be composed of terms from the van der Pol series \((x^6, x^5, x^4, \ldots)\) or from the Rayleigh series \((x^3, x^2, x^1, \ldots)\), separately or in combination. They also showed the viability, in addition to the well-known nonlinearity of Rayleigh and van der Pol, of a new type of series expansions they called \( \pi \)-mix series (even terms, \( x^2, x^4, \ldots \); odd terms, \( x^1, x^3, \ldots \)). Subsequent experiments showed that that catalogue was sufficient for constructing models that adequately capture the main kinematic features of various kinds of rhythmic and discrete movements (e.g., Beek et al., 1996; Beek et al., 1995; Mottet & Bootsva, 1999; Zaal, Bootsva, & van Wieringen, 1998).

In a previous experiment on the ski-simulator, Delignières et al. (1999) analyzed the movements of the platform with three groups of participants who were requested to oscillate, respectively, with amplitudes of 15 cm, 22.5 cm, and 30 cm, during four sessions. We showed that, in general, the oscillations of the platform were adequately captured by the following model:

\[ \ddot{x} + c_{30} x^3 + c_{50} x^5 + c_{70} \dot{x}^2 - c_{21} x^2 \dot{x} = 0, \quad (2) \]

which contains a nonlinear stiffness function \( c_{30} \) was negative and \( c_{50} \) positive, denoting a complex process of softening and hardening of the stiffness during the oscillation) and a van der Pol damping function (note that in those equations, one indexes the coefficients by using the W-method notation proposed by Beek and Beek, 1988, where \( c_{21} \) denotes the coefficient of \( x^2 \dot{x} \)).

In that experiment, we found a significant effect of the amplitude of the oscillations on the coefficients of the stiffness function, which was highly nonlinear at 22.5 and 30 cm and was almost linear at 15 cm. We also obtained an effect of practice, with a progressive linearization of the stiffness function over the four sessions. Concerning the damping function, we observed only one exception to the general van der Pol solution—a participant of the "30-cm group" whose behavior was consistently accounted for by a
Rayleigh function. Finally, we observed no effect of practice on the damping behavior either qualitatively (the nature of individual models remained unchanged over the four sessions) or quantitatively (no significant evolution in the coefficients of the function).

The main contribution of that experiment was to show that although the limit cycle modeling strategy is focused on the movements of the platform, it provides valuable information about the overall behavior. Results indicated that the stiffness function and its evolution could not be entirely explained by the physical characteristics of the apparatus and should be considered as a global index of the behavioral adaptation to the requirements of the task. On the other hand, the damping function gave important indications about the way in which participants injected energy within the cycle. The forcing strategy constitutes, according to Vereijken (1991), the key variable for the study of learning on the ski-simulator.

The results of that first experiment were nevertheless difficult to interpret in terms of learning. Clearly, practice led to a linearization of the stiffness function, and that result was expected in the present experiment. Concerning the van der Pol behavior, which was characteristic of most of the participants, it was difficult to conclude whether it constitutes the typical behavior of the beginner or instead reveals a first level of adaptation to the task. It is important to note that all participants had previous ski experience and, moreover, that they had benefited from two sessions of familiarization before the experiment. As such, from the beginning of the experiment they were able to reach the required amplitudes, which were well beyond the typical amplitudes reported by Vereijken (1991) for a first confrontation with the task (especially for the 22.5-cm and the 30-cm groups).

Vereijken (1991) showed that with practice, participants tended to progressively delay the moment of forcing after the platform had passed the center position. Such delayed injection of energy is consistent with a van der Pol-type damping, which could be representative of an intermediate level of adaptation, following the initial novice behavior in the course of learning. In one of her experiments, Vereijken (1991, chap. 4) showed that during the first trials, participants tended to force the apparatus in the first part of the oscillation, before the platform passed through the middle position. Such early injection of energy could be indicative of an underlying Rayleigh damping, which then could represent true novice behavior. With the aim of clarifying that point, we obviously decided to begin the measures from the very first trial on the task and also to confront participants with a more difficult version of the task (see the following) in order to possibly delay the adaptation process.

To summarize, our goal was to analyze the effects of practice on the ski-simulator's platform dynamics. Because Vereijken et al. (1997) suggested that learning proceeds in successive stages, characterized by qualitatively distinct coordinative structures, we hypothesized that such qualitative changes in global behavior should lead to qualitative or quantitative alterations, or both, in the composition of the dynamical model derived from the movements of the platform. The stiffness function was expected to present high nonlinearities at the beginning of the experiment and a progressive linearization with practice. Our hypotheses concerning the evolution of damping were derived from the analyses presented in the preceding paragraph. As suggested by the results of Vereijken (1991), a Rayleigh damping behavior was expected during the first trials, and then a transition to the van der Pol behavior observed in the Delignières et al. (1999) experiment. Our main concern was to characterize the dynamics of that transition and to analyze its possible relationships with changes in stiffness, movement frequency, or movement amplitude.

Method

Participants

Four men and one woman (mean age = 29.2 ± 6.3 years, mean weight = 71.6 ± 4.6 kg, mean height = 179.6 ± 3.5 cm) volunteered to take part in this experiment. Four of the participants were occasional skiers (with, on average, 4 days of practice per year), but none had previous experience on the ski-simulator. They signed a consent form and were not paid for their participation.

Experimental Device

The task was executed on a slalom ski-simulator (Skier's Edge Co., Park City, UT), which consisted of a platform on wheels that moved back and forth on two bowed, parallel metal rails (Figure 1). Two rubber belts fastened the platform to the rails and ensured that it regained its resting posi-

![FIGURE 1. The ski-simulator apparatus.](image-url)
tion in the middle of the apparatus after a forced deviation. The tension of the belts was controlled with a dynamometer at the beginning of each session and was adjusted so that participants could obtain a displacement of 4 cm of the platform from the central position by exerting a tangential force of 100 N.

To make the task more difficult, we replaced the two independent foot supports of the original apparatus with a 30-cm-wide board, in unstable balance over a sagittal rotation axis (Figure 2). Our apparatus could then be conceived of as a kind of mono-ski-simulator. For safety reasons, the participants' feet were not strapped. To help participants maintain their feet on the board, we fixed two wooden sticks at the board's lateral extremities and covered the upper surface with a nonskid material.

**Procedure**

The participants were instructed to learn to make cyclical sideways movements on the ski-simulator. They were given no demonstration; all they were told was to make their movements as ample and frequent as possible. To avoid a possible hiding of the passive markers (see the following section on data collection), we instructed the participants to keep their hands behind their back at all times. Finally, they were told to fix their eyes on a point located on the floor, 3 m in front of the apparatus.

Each session consisted of ten 1-min trials, with a 1-min break between trials. Each participant practiced for 3 sessions per week, over a period of 13 weeks, for a total of 39 sessions or 390 trials and a total practice time of 6 hr and 30 min. Finally, a retention test, consisting of ten 1-min trials, with a 1-min break between trials, was conducted 5 months after the end of the learning schedule.

**Data Collection**

The position of the middle point of the platform was measured by a potentiometer (Radiospares [Beauvais, France] 20-K resistance and .25% linearity) and was sampled at a frequency of 100 Hz. One revolution of the potentiometer reflected 12.5 cm of movement of the platform. The signal was recorded from the 15th to the 45th s of each trial. Data were stored on a personal computer for further analysis. During the measurements trials, we also equipped the participants with passive markers whose positions were recorded in three dimensions by a Vicon 370 (Oxford, England) motion analyzer, in order to characterize the evolution of coordination with practice. The motion analysis data are not included in this article but will be presented in a future contribution.

To avoid time-consuming treatments, and expecting that the most important changes would occur during the first weeks, we adopted a measurement schedule similar to that proposed by Vereijken (1991), with frequent measurements at the beginning of the experiment and a progressive spacing out following the advance of the protocol. The measurement schedule during the 39 learning sessions is summarized in Table 1. During the retention test, measurements were conducted on Trials 5 and 6 (the first 4 trials were considered as refamiliarization with the task).

**Data Reduction**

The position–time series were filtered at a cutoff frequency of 10 Hz with a dual-pass, second-order Butterworth filter.

![FIGURE 2. Details of the arrangement of the ski-simulator in the mono-ski version.](image)

<table>
<thead>
<tr>
<th>TABLE 1. Measurement Schedule During the 39 Learning Sessions</th>
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<tr>
<td>1–6</td>
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June 2003, Vol. 35, No. 2
The cutoff frequency was chosen following an analysis of the spectral composition of the time series, such that the essential characteristics of the signal were preserved.

We used a peak-finding algorithm to locate the reversal points of the movement (i.e., the points of maximal deviation, to the right or to the left). Cycle frequency (in Hertz) was defined as the inverse of the time between two successive right reversals. Cycle amplitude (in centimeters) was defined as the mean of the maximal deviations of the platform from the rest position at the right and left reversal points of the cycle. Means and standard deviations of amplitude and frequency were computed for each 30-s sample.

Each sample was then summarized in a normalized average cycle, according to the procedure adopted by Mottet and Bootsma (1999). The computation involved the following steps. First, the 30-s time series were segmented into half cycles that represented the motion from a reversal point to the following reversal point. By means of linear interpolation, we then normalized each half cycle by using 21 equidistant points. The points were then rescaled within the interval (−1, +1). The normalized half cycles beginning at the same reversal point were averaged point by point, and we obtained the normalized average cycle (42 points) by combining the back-and-forth normalized average half cycles. The first and second derivatives were computed from the normalized average cycle and then rescaled within the interval (−1, +1). As can be seen, our data were normalized both with respect to time (to allow the averaging of cycles) and with respect to amplitude.

Modeling Strategy

Our modeling strategy was based on the analysis of the normalized cycles, which were assumed to represent the dynamical organization that emerged in response to the task demands. We supposed that the limit cycle attractor did not change at the time scale of observation and that stochastic fluctuations at a microscale level constituted random noise, pushing the observed behavior around the average attractive pattern. Our method, in accordance with the principles developed by Beck and Beek (1988), combined qualitative graphical analyses, which enabled us to identify the nonlinear components underlying platform movements, and quantitative statistical procedures, which allowed us to estimate the magnitude of the respective contribution of each component and its changes with practice.

In a first step, we used Hooke’s plane representations (position vs. acceleration) for a direct assessment of the stiffness function (Delignières et al., 1999; Guiard, 1993; Mottet & Bootsma, 1999). The Hooke’s portrait shows a straight line for a perfect harmonic oscillator, and all tendencies to deviate from that straight line give valuable information on the nonlinear stiffness terms to include in the model. For example, an N-shape in the Hooke’s plane suggests a local softening-spring behavior, which could be accounted for by the inclusion of a negative cubic Duffing term (−x^3) in the model (Mottet & Bootsma, 1999). Previous analyses on ski-simulator data showed the necessity for adding a positive quintic term (x^5) in the stiffness function to account for a restoration of stiffness near the reversal points of the movement (see Delignières et al., 1999, Figure 1, p. 775).

The determination of relevant nonlinear damping terms was also based on graphical analyses. To isolate the contribution of nonlinear damping, we first performed a regression of −x against all previously identified linear and nonlinear stiffness terms and linear damping (x). The residual (RES/x) of that regression was assumed to reflect the contribution of nonlinear damping terms to behavior. Then, we applied the principles proposed by Beck and Beek (1988): We searched for van der Pol behavior by plotting the residual against x (in that case, a parabola was expected, revealing the presence of an x^2 term in the residuals) and for Rayleigh behavior by plotting the residual against x^2 (looking for parabola for x^2 even behavior and for N-shape for x^2 odd behavior; see Beck & Beek, 1988).

Our aim in performing those graphical analyses was to determine a minimal dynamical model containing a limited set of relevant terms. Then, we assessed the relative importance of each coefficient by using a stepwise multiple regression procedure of all relevant terms onto −x, as suggested by the original W-method (Beek & Beek, 1988). Note that another and faster method could be to directly use stepwise regression of all viable terms (i.e., x, x^3, x^4, x^5, x^6; see Beek & Beek, 1988) in order to select the terms to include in the model. Nevertheless, the general viability of such a dynamical model imposes specific sign constraints: In particular, the linear damping term must be negative to give rise to a limit cycle, and at least one of the nonlinear damping terms must be positive. Mottet and Bootsma (1999) noted that with stepwise regression of all possible terms in W(x, x), one is unable to take those sign constraints into account and is led mostly to inconsistent results (e.g., unstable models or nonsignificant linear damping). Hence, the use of graphical methods to qualitatively derive a minimal model constitutes a good preliminary step to the application of the W-method.

Those methods were applied on normalized average cycles, and one could wonder whether such normalization might have affected the shape of the limit cycles. Our aim in those analyses in normalized space was to render all coefficients in the model comparable and to clearly address the topological aspects of the limit cycle. Preliminary analyses showed that normalization affected the magnitude of the coefficients in the model, but the nature of the terms constituting the model remained unchanged (see also Mottet & Bootsma, 1999).

Finally, note that our approach remained strictly individual. As pointed out by K. M. Newell (1991), the essential features that could appear in the course of learning occur at
individualized time scales, and any averaging could obscure the true nature of the learning process. Our aim in our approach was, then, to examine the trends that are shared between individuals rather than to analyze the average trends of an experimental group.

**Results**

**Amplitude and Frequency**

The individual evolution of the amplitude of platform motion is reported in Figure 3. As can be seen, the amplitude of the oscillations was very small during the first trials (between 5 and 10 cm), with the exception of Participant 1, who seemed able to spontaneously oscillate around 20 cm from the first confrontation with the task onward. However, during the 1st week of practice (from Trial 1 to Trial 30), all participants reached a mean amplitude of about 30 cm. Amplitude growth appeared quite limited after that initial increase.

The individual evolution of frequency is reported in Figure 4. As can be seen, the shapes of the individual curves are quite different. Nevertheless, the curves presented, generally, a global increase of frequency during the course of the

![Figure 3](image-url)  
**FIGURE 3.** Individual evolutions of mean amplitude of platform movement over trials.
experiment. The only exception was Participant 3, who adopted an oscillation frequency of around 1.2 Hz throughout the experiment. Participant 1 presented a quite low frequency (= 1 Hz) at the beginning of the experiment, and that frequency tended to progressively increase, to about 1.4 Hz around the 150th trial (Session 15). The evolution of frequency for Participants 2, 4, and 5 was characterized by a quite abrupt shift, from a value of about 1.0 Hz to a value of about 1.4 Hz. The shift occurred between Trials 81 and 85 for Participant 2 (Session 8), between Trials 53 and 56 for Participant 4 (Session 5), and between Trials 100 and 121 for Participant 5 (Sessions 10–12). Finally, note in the figure that the very first trials were sometimes characterized by the adoption of very high frequencies: That was so for Participant 2, who oscillated at frequencies around 1.7 Hz during the first 2 trials. Then the frequency progressively decreased during the first 2 sessions, to reach approximately 1.0 Hz, as for most of the other participants. The frequency then remained more or less constant during 5 sessions, until an abrupt shift to 1.4 Hz occurred. A quite similar evolution, albeit with lower initial frequencies, was found for Participant 3.
Dynamical Modeling

Visual inspection of the Hooke portraits revealed in most cases a nonlinear stiffness function. A typical Hooke portrait, illustrating the common features we observed in most trials, can be seen in Figure 5. As in our previous analyses (Delignières et al., 1999), we needed to include a negative cubic Duffing term ($-x^3$) in order to account for the local decrease of stiffness symmetrically observable in the Hooke's portrait. In addition, a positive quintic ($x^5$) term had to be included to account for the final restoration of stiffness, close to each reversal point. As can be seen in Figure 5, right panel, those typical alterations of the Hooke's portrait were also present, albeit less pronounced, at the end of the experiment. We therefore decided to include the following stiffness function in the model for all measured trials:

$$g(x) = c_{10}x + c_{30}x^3 + c_{30}x^5,$$

where $c_{10}$ and $c_{30}$ being positive and $c_{30}$ negative.

The graphical scouting for nonlinear damping terms revealed that during the first part of the experiment, the motion of the platform was characterized by Rayleigh-type behavior. Figure 6, left panel, illustrates the typical graphical representation obtained during the first sessions, between $\dot{x}$ and RES. That kind of N-shape clearly suggested the need to include a $\dot{x}^2$ term in the damping function. In contrast, the graphical scouting indicated during the last part of the experiment the presence of a van der Pol behavior. As illustrated in the right panel of Figure 6, the plot of RES/$\dot{x}$ against $x$ contained a clear parabola, suggesting the inclusion of an $x^2\dot{x}$ van der Pol term in the damping function.

The Rayleigh behavior was consistently adopted by all participants, from the beginning of the experiment, for a minimum duration of 4 sessions (Participant 1) and a maximum of 8 sessions (Participant 3). A notable exception to that general result was Participant 2, who exploited a van der Pol behavior during the first 2 sessions. Nevertheless, that participant switched abruptly at the beginning of the subsequent sessions.
3rd session to a Rayleigh behavior, joining the common tendency. On the other hand, the van der Pol behavior was consistently exploited during a maximum of 11 weeks (Participant 2, from Session 7 to the end of the experiment) and a minimum of 6 weeks (Participant 3, from Session 21 to the end of the experiment). Between the initial Rayleigh stage and the final van der Pol stage, a number of consecutive trials appeared more difficult to characterize. During the transitory period, the Hooke's portraits seemed almost linear, and RES did not contain sufficient information to allow a valid assessment, at least by graphical methods, of the underlying damping function. Note, finally, that we never found any trace of π-mix behavior by using the graphical procedures proposed by Beek and Beek (1988).

In conclusion, the first graphical step allowed us to determine two minimal models that seemed able to adequately account for the dynamics of the platform motion: A Duffing + Rayleigh model for the initial stage of learning, that is,

\[ x + c_{10}x + c_{30}x^3 + c_{50}x^5 + c_{01}x + c_{03}x^3 = 0, \]  

and a Duffing + van der Pol model for the final stage,

\[ x + c_{10}x + c_{30}x^3 + c_{50}x^5 + c_{01}x + c_{21}x^2 = 0. \]

Evaluation of the Coefficients Within the Models

The quantitative assessment of the contribution of each term constitutes an important test for the validity of the models derived from the graphical analysis. As stated previously, a dynamical model should satisfy specific sign requirements to give rise to a limit cycle. Because multiple regression procedures are not subjected to sign constraints, the correspondence of the signs of the obtained coefficients with the basic requirements for limit cycle generation constitutes a good indication of the adequacy of the model. In addition, the quantitative assessment of the coefficients allowed the analysis of the evolution of the contribution of the linear and nonlinear components with practice.

Hooke's portraits allow a first global approach of the relative contribution of nonlinear terms within the underlying model. The amount of variance that can be attributed to simple harmonic motion can be measured by the coefficient of determination \( r^2 \) of the linear regression of position onto acceleration. The residuals of that regression measure the influence of the sum of all nonlinear terms. As such, one can assess the percentage of variance that is attributed to nonlinear components by using the quantity \( 1 - r^2 \), which could be considered as an index of harmonicity (Guillard, 1993; Nourrit, Lauriot, Deschamps, Caillou, & Delignières, 2000; Motter & Bootsma, 1999). The individual evolution of that quantity with practice is presented in Figure 7.

Generally, the contribution of nonlinear components decreased quite early during practice and reached a minimum value between the 50th and the 100th trials. Then, \( 1 - r^2 \) remained very low until the end of the experiment, suggesting a global linearization of the models. The evolution was not exactly identical among individuals, however. The contribution of nonlinear terms was generally lower for Participants 1 and 3 than for the other participants. During the 1st week, the mean percentages of variance explained by nonlinear components for those 2 participants were about 15.5% and 11.8%, respectively. In contrast, we obtained for Participants 4 and 5, 39.0% and 57.5%, respectively. Participant 2 presented quite a different profile, with a surprisingly high linearity during the first 2 weeks (6.1% of variance was explained by nonlinear components) and an abrupt increase of the contribution of nonlinear components from the beginning of the third session (36.7% of explained variance for that session). Then, that contribution decreased progressively and reached its minimal values during the ninth session.

The coefficients of the stiffness terms \( (c_{10}, c_{30}, c_{50}) \) were generally of expected signs \( (c_{10} \text{ positive and } c_{30} \text{ negative}) \). On some occasions, an inversion of the sign was observed, but in those cases the obtained coefficients were usually not significantly different from zero. The evolution of those coefficients with practice appeared closely coupled. Across all measured trials, high correlation coefficients between coefficient samples were found \( (r_{10-30} = -970, r_{30-50} = .925, \text{ and } r_{30-50} = -.985) \). Figure 8 presents the individual evolution of the coefficient \( c_{30} \). The evolution of the two other coefficients was similar.

As can be seen in Figure 8, \( c_{30} \) presented high negative values in the first part of the experiment, and then reached values near zero approximately from the 50th trial onward. The transition appeared quite abrupt for Participant 4 but more progressive for others. One could note that the initial \( c_{30} \) coefficients were lower (in absolute values) for Participant 3 than for the other participants. Finally, we observed some anomalous (positive or near-zero) values during the very first sessions, especially for Participant 2 during the two first sessions (see also Participants 4 and 5). The evolution of \( c_{50} \) was symmetrically identical. Coefficient \( c_{10} \) followed the same trend, with a final stabilization around 1.0.

In conclusion, and after a short period of adaptation for some participants, the stiffness function appeared highly nonlinear during the first part of the experiment. Practice led to a linearization of the stiffness function, which appeared established for every participant between the 50th and the 100th trial.

Concerning the damping function, the results of the multiple regression procedure generally confirmed the adequacy of the models derived from the graphical analyses, at least for the first Rayleigh and the final van der Pol stages. The signs of the obtained coefficients were always negative for the linear damping term and positive for the nonlinear Rayleigh or van der Pol term. We obtained only four exceptions (negative nonlinear damping coefficients), but in those rare cases, the coefficients were not statistically different from zero. The analysis also confirmed the exploitation of a van der Pol behavior by Participant 2 during the first 2 sessions. If we consider the only period during which all participants adopted for all measured trials a Rayleigh behav-
ior (Sessions 3 and 4), then mean $c_{03}$ was approximately 0.844 ($SD = 0.311$). Conversely, considering the final phase during which all participants adopted for all measured trials a van der Pol behavior (Sessions 21 to 39), mean $c_{21}$ was about 0.279 ($SD = 0.109$).

For the trials included in the transition stage, the two models were systematically tested. Generally, one of the two models produced a satisfying signs pattern, whereas the other did not. The obtained values for $c_{03}$ (Rayleigh) or $c_{21}$ (van der Pol) were lower than the corresponding values reported for the initial or final stages and were often nonsignificantly different from zero. At that level of observation (the averaged normalized cycles), the transition from the Rayleigh to the van der Pol behavior appeared through (a) a progressive decrease of the $c_{03}$ values; (b) the occasional emergence, generally for one isolated trial, of a van der Pol behavior; and (c) the progressive generalization of the van der Pol behavior, over all the measured trials. The first occasional occurrences of the van
der Pol behavior arose during Session 5 for Participants 1 and 2, during Session 9 for Participant 3, during Session 10 for Participant 5, and during Session 15 for Participant 4.

To visualize that transition through the evolution of a single metric, we forced the evaluation of the Duffing + Rayleigh model for all measured trials. Obviously, that procedure provided unsatisfactory results, in terms of signs requirements, when the underlying model was of the van der Pol type. In those cases, we systematically obtained a positive linear damping term ($c_{01}$) and a negative nonlinear damping term ($c_{02}$). When a van der Pol behavior is forced to the Rayleigh model, that procedure has the property of providing linear damping terms proportional in absolute values (but of opposite sign) to the original corresponding terms. Over 98 measured van der Pol trials reassessed by the forcing procedure, we obtained a correlation coefficient of $-0.937$ between the original linear damping terms and the forced corresponding terms. That property is related to the fact that Rayleigh and van der Pol functions act orthogonally in the phase space. Note, however, that the forcing procedure tends to overestimate, in absolute value, the linear damping term: Over the 98 measured van der Pol trials, the original mean $c_{01}$ was $-0.158$ ($SD = 0.093$) and the mean forced $c_{01}$ was $0.458$ ($SD = 0.282$).

We called the forced linear damping coefficient $c_{01}^{Rayleigh}$.
In summary, $c_{01}(\text{Rayleigh})$ allows a continuous description of the damping behavior, despite a qualitative evolution of the underlying models. Coefficient $c_{01}(\text{Rayleigh})$ is negative when the limit cycle is sustained by a Rayleigh behavior and positive for a van der Pol behavior. Finally, the absolute value of $c_{01}(\text{Rayleigh})$ gives an indication about the stability of the corresponding behavior: Coefficients $c_{01}(\text{Rayleigh})$ close to zero are indicative of unstable limit cycles, with large relaxation times, and, conversely, $c_{01}(\text{Rayleigh})$ far from zero (positive or negative) suggest more attractive trajectories in the phase space. The individual evolution of $c_{01}(\text{Rayleigh})$ is presented in Figure 9.

Despite individual differences in the evolution of $c_{01}(\text{Rayleigh})$, some common features were easily discernible, namely:

![Graph showing individual evolutions of $c_{01}(\text{Rayleigh})$ over trials for participants 1 to 5.](image)

**FIGURE 9.** Individual evolutions of the coefficient $c_{01}(\text{Rayleigh})$ over trials. One obtains that coefficient by forcing the Duffing + Rayleigh model ($\dot{x} + c_{10}x + c_{30}x^3 + c_{50}x^5 + \dot{c}_{01}x + c_{03}x^3 = 0$) on each averaged normalized cycle, using multiple regression analysis. The linear damping coefficient obtained with that forcing procedure, $c_{01}(\text{Rayleigh})$, is negative when the limit cycle is sustained by a Rayleigh behavior and is positive for a van der Pol behavior. Zero-crossings indicate a transition between behaviors.
• a first stage that was characterized by a highly stable Rayleigh behavior. That phase lasted approximately 50 to 70 trials. Generally, the first session gave inconsistent results, whereas the stabilization of the Rayleigh behavior was really effective from the second session. Note the particular evolution of \( c_{01}(\text{Rayleigh}) \) for Participant 2 at the beginning of the experiment, confirming the adoption of a quite stable van der Pol behavior during the two first sessions.

• a final stage that was characterized by the stabilization of the van der Pol behavior. The stabilization was effective from the 90th trial for Participant 2 but was not really evident before the 300th trial for Participant 3. The other participants seemed to stabilize the van der Pol behavior between Trials 150 and 200.

• a transition stage between the Rayleigh and the van der Pol behaviors, which seemed to be characterized by low absolute values of \( c_{01}(\text{Rayleigh}) \) (in other terms, a low stability of the corresponding behavior) and by the alternating exploitation, from one trial to the other, of the two concurrent damping behaviors.

As stated previously, the damping coefficients during that stage of transition were often not significantly different from zero. In other words, the platform motion, at the level of the averaged normalized cycles, looked like a harmonic oscillator, a result that was obviously not realistic. It is important to keep in mind that the averaged normalized cycles represented the average behavior of about 30 successive cycles. To analyze the nature of the transition in more detail, we performed for each participant a cycle-to-cycle analysis on a set of selected trials. Representative results, as obtained from three trials of Participant 1, are shown in Figure 10. Generally, during the initial Rayleigh stage, each trial was composed of successive cycles homogeneously sustained by a Rayleigh damping behavior. During the van der Pol stage, a symmetrical result was described for the late trials. Generally, \( c_{01}(\text{Rayleigh}) \) presented a lower cycle-to-cycle variability during that final phase than during the initial one (see, for example, Figure 10, right and left panels). In contrast to the homogeneity of the previously described behaviors, the stage of transition appeared characterized by the alternating exploitation of the two concurrent damping behaviors from cycle to cycle. As illustrated in Figure 10 (central panel), \( c_{01}(\text{Rayleigh}) \) during the trials of that transition stage presented frequent alternations between (significantly) negative and positive values.

For professional reasons, Participant 5 was unable to take part in the retention test proposed 5 months after the end of the learning period. The mean amplitude reached by the 4 remaining participants during the two measured trials was therefore 37.7 cm (SD = 2.3), and the mean frequency was 1.4 Hz (SD = 0.1). Those mean values were almost identical to those observed during the two last measured trials of the learning period (amplitude = 36.0 cm, SD = 3.4; frequency = 1.4 Hz, SD = 0.1). Most important, the averaged normalized cycles for the 4 participants were clearly accounted for by the Duffing + van der Pol model. Those models were quantitatively similar to those obtained at the end of the learning period (\( c_{01} = -1.08, SD = 0.90 \), for the end of the learning period, vs. \( c_{01} = -0.311, SD = 0.228 \), for the retention trials; \( c_{01} = -1.74, SD = 0.35 \), for the end of the learning period, vs. \( c_{01} = -0.216, SD = 0.068 \), for the retention trials; \( c_{21} = 0.246, SD = 0.049 \), for the learning period, vs. \( c_{21} = 0.304, SD = 0.066 \), for the retention trials).

**Discussion**

Our main aim in this experiment was to study the qualitative reorganizations of behavior that appear during the acqui-

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**FIGURE 10.** Cycle-to-cycle analysis of \( c_{01}(\text{Rayleigh}) \) coefficients, during the 37th, 91st, and 335th trials of Participant 1. Those trials were representative, respectively, of the Rayleigh stage, the transition stage, and the van der Pol stage. The coefficients that did not differ from zero are plotted as white circles.
sition of a complex motor skill. We used a sport-like task, performing slalom-like movements on a ski-simulator, because it has previously been suggested that the learning of that task is characterized by qualitative reorganizations (Vereijken, 1991). In our analyses, we focused on the modeling of the motion of the platform of the apparatus by using the methods developed by Beek and Beek (1988) and Mottet and Bootsma (1999). We recently showed that those methods provide valuable information about the global behavior and about its adaptation to the prevailing task constraints (Delignières et al., 1999).

The results of the present experiment evidenced parallel evolutions for our 5 participants, with an initial stage characterized by a low oscillation frequency, a highly nonlinear stiffness function, and a Rayleigh damping function, and a final stage characterized by a higher frequency, the linearization of stiffness, and a van der Pol damping function. The transition from the initial stage to the final one appeared as a gradual rather than an abrupt process.

**Spontaneous Coordination**

Some divergent results were found during the very first sessions (especially for Participant 2); however, the first type of behavior was systematically adopted by all participants at the beginning of the experiment. Such between-participants consistency was previously observed in similar experiments (see, for example, Delignières et al., 1998) and suggests that the behavior of beginners in a given task is governed by common coordination tendencies. In other words, and despite possible interindividual differences, novices in a given task share quite similar intrinsic dynamics. In a number of experiments focused on bimanual coordination, similar results have been reported—spontaneous prevalence of the in-phase and antiphase modes of coordination between the two hands (Kelso, 1995; Kelso, Holt, Rubin, & Kugler, 1981). Nevertheless, it seems important to note that such interindividual consistency could also arise in more complex tasks involving significantly more degrees of freedom. As such, learning cannot be conceived of as the transition from disorder to order but as the transition from an initial order to a higher order, more specifically adapted to the task’s constraints. The first stage we observed in the present experiment could then be considered as the novice behavior, a quite stable initial behavioral solution consistently adopted by beginners, and which constitutes the basis on which all subsequent learning will occur (Swinnen et al., 1997).

The results of the present experiment confirmed a finding previously highlighted by Delignières et al. (1998): Those initial behaviors can be characterized, in some tasks, by a high resistance to change. The Rayleigh damping behavior was exploited by most participants during 5 sessions (i.e., 50 min of cumulative practice), a duration that exceeds the total duration of most previous learning experiments. Note, nevertheless, that that first novice behavior cannot be considered as inefficient. The major improvements, in terms of oscillation amplitude, were observed during the initial stage, from Sessions 1 to 3. The same observation was made by Delignières et al. (1998) in their experiment on parallel bars. That remark suggests, on the one hand, that in the present task amplitude is not a good criterion for assessing expertise, and on the other, that the transition from the first stage to the second is not motivated by a lack of efficacy.

**Novice Dynamical Model**

During the first stage, the stiffness function appeared highly nonlinear. The system as a whole behaved as a softening, then hardening spring. It is important to keep in mind that the stiffness function determines the frequency of the oscillator. The $x^4$ term, with a negative coefficient, results in a local slowing down, whereas the $x^3$ term allows a restoration of the initial stiffness in the vicinity of the reversal points. During that first stage, the $c_4$ coefficient was, in absolute value, higher than the $c_3$ coefficient and entailed a detuning of the system, which thus oscillated at lower frequencies than its eigenfrequency (Beek, Turvey, & Schmidt, 1992).

The main incidence of the local alteration of stiffness was to provide participants with a prolonged dwelling time when approaching the reversal points of the movement. Mottet and Bootsma (1999) observed a similar dynamics of the stiffness function in Fitts’s reciprocal tapping tasks. The stiffness was quasi-linear for easy tasks; but for difficult tasks, they had to add a negative nonlinear Duffing term to account for its softening dynamics. The local decrease of stiffness allowed participants to benefit from a dwelling time in the vicinity of the target and then to ensure an acceptable accuracy while keeping the global oscillation frequency as high as possible.

In a similar fashion, novices on the ski-simulator need time to manage the reversal points of the oscillations. After novices have forced the platform far away from its central position, they should readjust their behavior in order to optimally benefit from the restoring of the energy stored in the rubber belts. That adjustment requires complex postural adaptations, which could be particularly difficult to achieve on the mono-ski-simulator used in the present experiment. The beginner cannot achieve those postural adaptations when subjected to the system’s eigenfrequency, which can be supposed as mainly determined by the stiffness of the rubber belts. The nonlinear stiffness function we observed provided participants the dwelling time they needed, while preserving the benefit of the restoring force of the apparatus.

It is important to note at this point that the mechanical stiffness of the rubber belts, at least within the range of amplitudes used in the present experiment, was strictly linear (Delignières et al., 1999). As such, the nonlinear stiffness function obtained during the first part of the experiment cannot be explained by the mechanical properties of the springs; instead, it reflected the behavior of the participant–apparatus system. In the same vein, one could suppose that the linearization of the stiffness function observed later in practice could have been caused by the fact that with large amplitudes, more energy might be stored in the rubber belts, and that the physical properties of the apparatus then contributed.
more to the overall coordination dynamics. The results of Delignières et al. (1999), showing that the linearization of the stiffness function was faster for the smallest amplitude (15 cm), clearly contradicted that hypothesis.

One could think that the Rayleigh damping behavior contributed also to the availability of dwelling time for the management of the following reversal. Rayleigh damping is characterized by an early occurrence of peak velocity, during the first part of the trajectory from one reversal point to the next. As such, the second part of the trajectory, after the passage through the central position, presents a relative slowing down. Motteb and Bootsma (1999), in their experiment on Fitts’s reciprocal tapping tasks, showed that the Rayleigh damping behavior was particularly adopted when the tasks were difficult. On the contrary, for the easiest tasks, the numerical assessment of their Rayleigh model led to the obtaining of positive linear damping coefficients, suggesting that in those cases, the damping behavior was rather of van der Pol type. The Rayleigh damping behavior in those two experiments appeared to play an essential role when the system needed time in the final part of the trajectory for managing the movement reversal.

Transition to van der Pol Behavior

The present interpretation does not justify, nevertheless, the necessity to bifurcate, at a given point, to a van der Pol model. A well-known property of those two damping functions is that they induce different relations between amplitude and frequency. In the case of van der Pol damping, amplitude is independent of frequency. On the contrary, in a Rayleigh oscillator an increase of frequency leads to a decrease in amplitude.

We think that those specific properties constitute a determinative point for the transition from the Rayleigh model to the van der Pol model. When participants become able to satisfactorily master the reversal points of the movement, the local slowing down (controlled by the x^2 term) is no longer necessary. Then the c_30 coefficient decreases (see Figure 8), determining an increase of frequency (Figure 4). We previously noted that at that moment of the experiment, participants have already reached large amplitudes. The typical amplitude–frequency relationship of the Rayleigh oscillator does not allow the maintenance of such large amplitudes with an increase of frequency (recall that mean frequency roughly increased from 1.0 Hz to 1.4 Hz). Consequently, the adoption of a damping behavior that preserved amplitude under high frequencies appeared necessary.

Behavioral Reorganization as Phase Transition

The present discussion leads to an important question that should be necessarily addressed to each learning experiment performed since the appearance of the study by Zanone and Kelso (1992): Can we consider the qualitative reorganizations of behavior that occur during learning as phase transitions, in the sense given to that concept by nonlinear dynamical systems theory (Kelso, 1995)? A phase transition or bifurcation is defined as an abrupt qualitative change in the behavior of the system. Underlying a modification of the intrinsic dynamics of the system, a bifurcation is determined by the evolution of a so-called control parameter. For example, in the well-known bimanual task used by Kelso et al. (1981), a phase transition occurs from the antiphase to the in-phase pattern when the frequency of oscillation (the control parameter), driven by means of an auditory metronome, increases beyond a critical value. Such a phase transition is typically preceded by an increase of the variability of the behavior of the system.

At first glance, the transition we observed from the Rayleigh model to the van der Pol model was not abrupt (Figure 9) but appeared rather progressive. That progressiveness could be considered as a major obstacle to considering that phenomenon as a phase transition. One must perform a deeper analysis of the evolution of behavior during the transition to determine more precisely its nature. During the transition stage, the damping coefficients, at the level of the normalized averaged cycles, were mostly nonsignificant. Nevertheless, the cycle-to-cycle analysis showed, during those trials, a continuous alternation between the two types of damping behaviors. Considering that the Rayleigh and van der Pol functions tend to act orthogonally in phase space (Motteb & Bootsma, 1998), the obtaining of quasi-harmonic averaged cycles for those trials is not so surprising. One could also note that the damping coefficients (c_01, c_03, c_21) during those transition trials were generally lower, in absolute values, than during the initial Rayleigh stage or the final van der Pol stage. Those results are typical of a bistable regime, where two concurrent attractors are simultaneously available but with low intrinsic stabilities (considering the magnitude of the damping coefficients as revealing the intrinsic stability of the limit cycle).

Then we observed a progressive transition from a monostable regime (Rayleigh), to another monostable regime (van der Pol), with an intermediate bistable regime. The so-called saddle-node bifurcation is known to generate such phenomena (Diedrich & Warren, 1995; Kelso, 1995). Consider the following potential:

\[ V(x) = ax^4 + bx^2 + kx, \] (6)

where \( x \) represents a relevant order parameter of the system. In that potential function, \( a \) and \( b \) are of opposite signs, so that two stable states can be generated. The \( k \) term represents a control parameter that specifies the relative availability of those two potential stable states. The evolution of that potential function for \( a = 1/4, b = -1/2 \), and \( k \) varying from -1 to 1 is illustrated in Figure 11. Consider the system behavior as a function of increasing \( k \). As can be seen, for \( k = -1 \), a unique stable solution is present in the attractors landscape. The increase of \( k \) generates qualitative changes in the potential:

The initial attractor loses its stability progressively, and at \( k = -(4/27)^{1/2} \), a second stable solution appears, which will coexist with the initial solution until \( k = (4/27)^{1/2} \). Between those two \( k \) values, the system exhibits a bistable regime, with two coalescing solutions. For \( k \) close to 0, the two attractors pos-
ssez an equivalent, albeit low, stability. Finally, when $k$ goes beyond the critical value of $(4/27)^{1/2}$, only one stable fixed point remains in the system.

Diedrich and Warren (1995) used the saddle-node bifurcation to model the transition from walking to running that arises in locomotion when locomotion speed increases. In that case, the transition is abrupt, as is theoretically predictable for a continuous and linear increase of $k$. Nevertheless, the evolution of $k$ could also be conceived of as nonlinear, with a prolonged stay around values near zero. In that case, the bistable regime could persist during a more or less prolonged time interval, and a continuous alternation between the two coexisting stable solutions could appear because of the presence of stochastic forces in the system. Such alternation could also arise from intermittency dynamics, which characterize the behavior of low-dimensional deterministic systems near saddle-node bifurcations (Pomeau & Manneville, 1980).

Such a model could account for the major features observed in the present experiment: (a) the presence of two qualitatively different stable states, in the initial and the final stage of learning; (b) the loss of stability of the initial state, revealed by the decrease, in absolute values, of the damping coefficients; (c) the coexistence of two attractive solutions during the transition stage, with low stabilities and frequent alternations; and (d) the final stabilization of the second solution.

That hypothesis is obviously contradicted by the fact that the transition did not have the abrupt character classically expected in dynamical systems theory. But, can one expect an abrupt behavioral change during learning? The phase transitions observed in bimanual coordination (Kelso, 1995) or in locomotion (Diedrich & Warren, 1995) arise between preexisting stable solutions. In other words, the target pattern is potentially immediately available. An abrupt transition, in the case of learning, should correspond to a kind of insight learning, a rarely observed phenomenon in complex skill acquisition. The pattern to be learned is not immediately available but should be constructed during the transition. As such, we think that learning transitions should generally appear gradual, at least when considering complex skill acquisition.

Note that we observed a quite abrupt transition for Participant 2 (see Figure 9). The transition was also precocious, as compared with those of the other participants. The differences could be related to that participant’s surprising exploitation of a typical expert behavior during the first two sessions. For unidentified reasons, the participant appeared to possess before the experiment a kind of (transferred) experience on the task. That finding could explain a relative availability of the van der Pol behavior in the initial repertoire of Participant 2 and the shortness of the transition to the final behavior.

A remaining problem is the identification of the control parameter that allows the transition to occur. Our results suggested that the increase in frequency associated with the large amplitudes reached by participants played a major role in the destabilization of the initial pattern and determined the beginning of the transition stage. One could also think about practice, or experience, as a candidate control parameter. The latter possibility was contradicted, nevertheless, by the transitory adoption of a van der Pol behavior by Participant 2 during the two first sessions. Moreover, the transitory adoption of that behavior was accompanied by a high oscillation frequency. That observation supports the idea that the van der Pol behavior was induced more by frequency increase than by the cumulated amount of practice. Nevertheless, the hypothesis was challenged by the results of Delignières et al. (1999), who showed that most participants adopted a van der Pol damping even under restricted amplitudes and low frequencies. One must perform further experiments, manipulating oscillation frequency, to elucidate the respective roles of amplitude, frequency, and practice in the evolution of oscillatory behavior on the ski-simulator.

**Behavioral Reorganization as Parametric Evolution**

The interpretation in terms of phase transition remains questionable, however, because we were unable in this experiment to find evidence for the typical signatures of such a phenomenon (critical fluctuations, increase of relaxation time, and hysteresis). An alternative hypothesis could be to consider a more complex model, including simultaneous Rayleigh and van der Pol terms (Equation 7).
\[ x + c_{10}x + c_{20}x^2 + c_{50}x^3 + c_{51}x + c_{05}x^3 + c_{21}x^2 = 0. \]

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Kay et al. (1987) adopted such a hybrid model to account for the observed relationships between frequency, amplitude, and peak velocity in single and bimanual rhythmic movements (see also Beek et al., 1995, and Haken, Kelso, & Bunz, 1985). In the present experiment, the novice and expert behaviors could be considered as two expressions of the same model, with Rayleigh dominance for the former and van der Pol dominance for the latter. As such, the transition from the initial stage to the final stage could be seen as the progressive parametric evolution of a single model. Motter and Bootsma (1999) suggested that kind of progressive adaptation in their dynamical analysis of Fitts's tapping task. As they pointed out, the behavioral dynamics could be viewed as the combined influence of Rayleigh and van der Pol terms, with van der Pol being stronger than Rayleigh for easy tasks and with Rayleigh becoming more important for difficult tasks.

However, we never found in our graphical analyzes any trace of van der Pol behavior during the initial Rayleigh stage (excluding the very first trials) or of Rayleigh behavior during the final van der Pol stage. We also tried to test the hybrid model of Equation 7 by using multiple regression. Generally those attempts led to inconvenient results such as, in particular, unexpected positive linear damping coefficients.

Nevertheless, the W-method we used presents some limitations for the assessment of damping coefficients. As pointed out by Motter and Bootsma (1999), the W-method can only assess the sum of the combined influences of the Rayleigh and van der Pol terms. Moreover, such empirical limit cycles do not contain sufficient information for a reliable estimation of dissipation coefficient (Beek et al., 1995). Investigators should use more powerful methods to test the relevance of the hybrid model, for example, perturbation experiments that could more directly reveal the damping components of the oscillatory motion (Eisenhammer, Hubler, Packard, & Kelso, 1991).

Learning Transition and Task Difficulty

A final question remains: Why did we obtain in our previous experiment (Delignières et al., 1999) with novice participants a van der Pol damping behavior that seemed in the present work typical of an expert stage? As noted in the introduction, the participants of the previous experiment benefited from two sessions of familiarization before the beginning of the experiment; we provided those sessions to allow them to reach the required amplitudes from the first experimental session. On the other hand, the previous experiment was realized with the original version of the simulator, with an independent articulated support for each foot. One could suppose that with the easier version of the task, the transition to the van der Pol behavior could arise earlier in practice and that two sessions of familiarization could be sufficient to allow participants to surpass their initial Rayleigh damping behavior.

That hypothesis was recently tested by Nourrit (2000), who performed an experiment in which novices practiced for six sessions on the original ski-simulator. The results showed that the transition from the Rayleigh behavior adopted by all participants from the first trial to the van der Pol behavior appeared from one participant to the other between the second and the fourth session. Such early qualitative reorganizations of behavior were previously documented by Vereijken (1991) in her experiments on the ski-simulator. Vereijken observed that during the first trials, the forcing of the apparatus was initiated during the first part of the trajectory of the platform, before the ski-simulator platform had passed through the central position. After some 1-min trials of practice, the forcing time was delayed after the central position, in the second part of the trajectory. One could suppose that there is a direct relation between the nature of the damping function underlying the limit cycle and the time of forcing. We previously evoked the early occurrence of peak velocity for the Rayleigh oscillator. Conversely, peak velocity appears later for the van der Pol oscillator, in the second part of the trajectory. That hypothesis was successfully tested by Nourrit (2000), who showed that during the Rayleigh stage, the forcing was initiated when the platform reached an average phase of 62°, as compared with 95° during the van der Pol stage (note that the passing through the central position corresponded to a 90° angle phase). Those results suggest that during her experiments, Vereijken (1991) obtained transitions similar to those we observed, from a Rayleigh to a van der Pol model. More generally, those experiments showed that the resistance to change of the spontaneous modes of coordination advocated by Delignières et al. (1998) could strongly differ between tasks.

Concluding Remarks

The results of the present experiment provide reasonable evidence for a qualitative difference between the behavior adopted by participants during the first trials of the experiment and a more skilled behavior adopted after a number of practice sessions. The novice behavior, which exploited a highly nonlinear stiffness function and a Rayleigh damping function, allowed participants to preserve a dwelling time close to the extrema of the oscillations in order to control the postural adjustments necessary to manage the motion reversals. Later in practice, the skilled behavior characterized by a van der Pol damping function seemed necessary to simultaneously produce high frequencies and large amplitudes. Those behaviors seemed to qualitatively differ, corresponding, respectively, to different limit cycle organizations.

The present modeling approach allows one to go beyond the classical approach to learning, based on the analysis of outcome data, through a deep analysis of coordination dynamics and its evolution. It provides fruitful insights concerning the causes underlying the adoption of a given behavior or the transition to another coordination. The present
results offer an interesting illustration of the installation of a new behavior during the learning process. The first trials on a new task are marked by the adoption of an initial behavior that seems to emerge from the intrinsic dynamics of the system. Practice leads to the replacement of the initial pattern by a new behavioral organization, which provides a more efficient way to perform the task. The exploitation of the initial behavior appears quite durable (see also Delignières et al., 1998) and seems essential for installing the required conditions of the subsequent substitution (as in the present experiment, the necessary increase in frequency and amplitude). That observation has important practical implications and suggests a reconsideration of the role of the initial patterns of coordination in the learning process.

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Submitted January 14, 2002
Revised June 6, 2002